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$$= 2x^2(1-x)^{-2} + x(1-x)^{-1} = x(1+x)(1-x)^{-2} = \frac{n^p+1}{(n^p-1)^2}$$

where we must have  $|x| < 1$ .

Also solved by Henry Heaton, A. H. Holmes, and G. B. M. Zerr.

### CALCULUS.

217. Proposed by Professor F. ANDEREGG, Oberlin College, Oberlin, Ohio.

$$\text{Find } \lim_{n \rightarrow \infty} \frac{1}{n} \sqrt[n]{(n+1)(n+2)\dots(2n)}.$$

I. Solution by the PROPOSER.

$$\text{Let } x = \lim_{n \rightarrow \infty} \frac{1}{n} \sqrt[n]{(n+1)(n+2)\dots 2n}$$

$$= \lim_{n \rightarrow \infty} \sqrt[n]{\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\dots\left(1 + \frac{n}{n}\right)}.$$

$$\text{Then } \log x = \lim_{n \rightarrow \infty} \frac{1}{n} \log \left[ \left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\dots\left(1 + \frac{n}{n}\right) \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{\lambda=1}^{\lambda=n} \log \left(1 + \frac{\lambda}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{\lambda=1}^{\lambda=n} \left( \frac{\lambda}{n} - \frac{\lambda^2}{2n^2} + \frac{\lambda^3}{3n^3} - \dots \right)$$

$$= \lim_{n \rightarrow \infty} \sum_{\lambda=1}^{\lambda=n} \sum_{\kappa=1}^{\kappa=n} (-1)^{\kappa-1} \frac{\lambda^\kappa}{\kappa n^{\kappa+1}}.$$

If the method of differences is used for  $\sum_{\lambda=1}^{\lambda=n} \lambda^\kappa = 1^\kappa + 2^\kappa + 3^\kappa + \dots$ , the  $\kappa$ th series of differences is

$$\begin{aligned} (\kappa+1)^\kappa - \binom{\kappa}{1} \kappa^\kappa + \binom{\kappa}{2} (\kappa-1)^\kappa - \binom{\kappa}{3} (\kappa-2)^\kappa + \dots \\ + (-1)^{\kappa-1} \binom{\kappa}{\kappa-1} 2^\kappa + (-1)^\kappa 1^\kappa \equiv \kappa!. \end{aligned}$$

The  $(\kappa+1)$ th series is

$$(\kappa+2)^\kappa - \binom{\kappa+1}{1} (\kappa+1)^\kappa + \binom{\kappa+1}{2} \kappa^\kappa - \dots + (-1)^\kappa \binom{\kappa+1}{\kappa} 2^\kappa + (-1)^{\kappa+1} 1^\kappa \equiv 0,$$

$\kappa$  being a positive integer.

If the first given number is represented by  $a$  and the successive differences by  $d_1, d_2, \dots$

$$S_{n,\kappa} = \binom{n}{1} a + \binom{n}{2} d_1 + \binom{n}{3} d_2 + \dots + \binom{n}{\kappa+1} d_\kappa.$$

First  $\lim_{n \rightarrow \infty} \frac{1}{\kappa n^{\kappa+1}} S_{n, \kappa}$  must be sought.

Only in the last term of  $S_{n, \kappa}$   $n$  appears in  $\kappa+1$  factors, therefore the preceding terms disappear, and

$$\lim_{n \rightarrow \infty} \frac{1}{\kappa n^{\kappa+1}} S_{n, \kappa} = \lim_{n \rightarrow \infty} \frac{1}{\kappa n^{\kappa+1}} \left[ \frac{n(n-1)(n-2) \dots (n-\kappa)}{(\kappa+1)!} \kappa! \right] = \frac{1}{\kappa(\kappa+1)}.$$

$$\text{Therefore } \lim_{n \rightarrow \infty} \sum_{\lambda=1}^{\lambda=n} \sum_{\kappa=1}^{\kappa=n} (-1)^{\kappa-1} \frac{\lambda^{\kappa}}{\kappa n^{\kappa+1}} = \sum_{\kappa=1}^{\kappa=\infty} (-1)^{\kappa-1} \frac{1}{\kappa(\kappa+1)}$$

$$= \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} + \dots = (1 - \frac{1}{2}) - (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) - \dots$$

$$= 2(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots) - 1 = 2 \log 2 - 1 = \log \frac{4}{e}.$$

$$\text{Hence, } \log x = \log \frac{4}{e}; \text{ and } x = \lim_{n \rightarrow \infty} \frac{1}{n} {}^n\sqrt{[(n+1)(n+2) \dots 2n]} = \frac{4}{e}.$$

II. Solution by S. A. COREY, Hiteman, Iowa.

$$\text{Evidently, } \frac{1}{n} {}^n\sqrt{[(n+1)(n+2) \dots (2n)]}$$

$$= \frac{1}{n} {}^n\sqrt{n^n (1 + \frac{1}{n})(1 + \frac{2}{n}) \dots (2)}$$

$$= {}^n\sqrt{(1 + \frac{1}{n})(1 + \frac{2}{n}) \dots (2)} = s \text{ (say).}$$

$$\text{Therefore, } \log s = \frac{1}{n} [\log(1 + \frac{1}{n}) + \log(1 + \frac{2}{n}) + \dots + \log 2].$$

Letting  $dx = 1/n$ , we have,

$$\lim_{n \rightarrow \infty} \log s = \int_1^2 \log x \, dx = 2 \log 2 - 1, \text{ or } s = \frac{4}{e}.$$

Also solved by Henry Heaton, and J. Scheffer. Several incorrect solutions were received.

239. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A thread makes  $n$  ( $=30$ ) equidistant spiral turns around a rough cone whose altitude is  $h$  ( $=10$  feet), and radius of base  $r$  ( $=11$  inches). How far will a bird fly in unwinding the thread if the part unwound is at all times perpendicular to the axis of the cone?